

# QAA Enhancement Theme – Introduction to Mathematical Concepts in Geophysics

## Introduction

We teach GL2511 “Introduction to Geophysics” to Level 2 Geology students, many of whom have a poor to non-existent background in basic mathematics and physics. Nevertheless, we find it imperative that some understanding of the mathematical formulation of principles essential to geophysics be achieved in this course. This is a major challenge for some students. Many students do not have the tools for us to get the message across of how mathematics can be used to formulate a problem with a solution that provides (geo)physical insights. We think that with an opportunity to refresh skills in basic mathematics many students will derive a much enhanced appreciation of the geophysical materials covered in lectures and exercises. In some cases we even expect that the recognition of the simple elegance of some of the geophysical concepts as applied could stimulate some students to adopt a more quantitative approach in their future geological studies. Ad hoc feedback from students in this year’s cohort suggests that there is an interest and willingness among them to engage in web-based learning modules designed specifically around course materials were these available.

## Aims and Objectives

We envision a set of MyAberdeen-based, self-learning Excel modules for our students to access, covering some elementary mathematical concepts and applications, corresponding with the material being dealt with in the course. These would comprise simple, illustrated (mainly graphs) exercises, with drag and drop answers (or otherwise, as appropriate), with prompts and customised feedback as “leading questions” for incorrect answers. Topics to be covered (and relevant course topics) include (i) trigonometric functions and simple planar geometry (seismic ray-paths in the Earth); (ii) 1-D functions, equations of lines and curves, plots (seismic phase travel-time curves); (iii) basics of differentiation and integration; derivatives and second derivatives of simple functions in 1-D.

## Projects Outcomes

Initially, we will compile statistical information on student participation, augmented by (non-assessed) online quizzes. The eventual outcome, which will be driven by the success of the project as based on initial feedback, will be to streamline and iteratively enhance the presentation of the quantitative geophysical and continuum mechanics course materials, providing a more rounded theoretical basis for those materials and, accordingly, an enhanced understanding of their usefulness and general applicability. We expect that enhanced modal skill level of our students will propagate into subsequent teaching years in geology. The course in 2013-2014 will have some 100+ students; we expect that the vast majority of these will make use of the enhancement if available.

Sphere 1		Conditions
Gravitational constant G (m <sup>3</sup> /kg s <sup>2</sup> )	6.674E-11	
Density contrast Δρ (kg/m <sup>3</sup> )	1000	TRUE
Radius R (m)	800	TRUE
Depth to the center of the sphere z (m)	850	TRUE
Volume V (m <sup>3</sup> )	2.145E+09	

  

Sphere 2		Conditions
Gravitational constant G (m <sup>3</sup> /kg s <sup>2</sup> )	6.674E-11	
Density contrast Δρ (kg/m <sup>3</sup> )	600	TRUE
Radius R (m)	800	TRUE
Depth to the center of the sphere z (m)	1000	TRUE
Volume V (m <sup>3</sup> )	2.145E+09	

  

$\rho_2 - \rho_1 = \Delta\rho$

**Range of values for the model:**  
 Δρ - change between 400 and 1000  
 R - change between 500 and 1000  
 z > R - depth to the centre of the sphere must be equal or greater than the radius!  
 All conditions **must** be TRUE.

  

Change values of Δρ, R, z in the orange cells to see how the gravity profiles of the spheres change. Can you achieve the same gravity profile for both spheres for different values of Δρ, R, z?  
**Note:**  
 Start with the same values of Δρ, R, z (i.e. Δρ = 600; R = 800; z = 1000) for both spheres so their profiles overlap. Change the values by small increments within given conditions to see how the profiles change.

  

First derivative of the function  $g_{z(x)}$   

$$\theta_{z(x)} = \frac{dg_{z(x)}}{dx} = \frac{\text{change of slope } g_{z(x)}}{\text{unit of length}}$$
 This function represents the instantaneous rate of change of the function  $g_{z(x)}$ . It shows how fast or slow the values change as the point P moves along the horizontal axis. The values are constant for areas of constant slope and show the most variation where slope changes quickly.

  

Second derivative of the function  $g_{z(x)}$   

$$\theta_{z(x)}'' = \frac{d\theta_{z(x)}}{dx} = \frac{\text{change of slope of } \theta_{z(x)}}{\text{unit of length}}$$
 This function represent the instantaneous rate of change of the function  $g_{z(x)}$  (first derivative).

## Discussion

We will carry out a baseline survey/assessment of our students at the outset of the GL2511 course (which begins in January 2014), the development of which will be part of the project, and again at the end of the course (May 2014). Over a longer term, we expect that the enhanced mathematical skills of our Level 2 students will be evident in most (geology) courses delivered in levels 3 and 4 and we will establish a mechanism within those courses to track performance in selected modules in them as next year’s (2013-2014) Level 2 cohort progresses through them, as compared with results mined from course records in academic years 2011-2012, 2012-2013 and 2013-2014.

Inclined Thin Rod 1		Conditions
Gravitational constant G (m <sup>3</sup> /kg s <sup>2</sup> )	6.674E-11	
Density contrast Δρ (kg/m <sup>3</sup> )	500	TRUE
Cross-section A (m <sup>2</sup> )	500000	TRUE
Inclination α (°)	45	TRUE
Length of rod L (m)	2000	TRUE
Height of rod top z (m)	50	TRUE

  

Inclined Thin Rod 2		Conditions
Gravitational constant G (m <sup>3</sup> /kg s <sup>2</sup> )	6.674E-11	
Density contrast Δρ (kg/m <sup>3</sup> )	500	TRUE
Cross-section A (m <sup>2</sup> )	500000	TRUE
Inclination α (°)	90	TRUE
Length of rod L (m)	2000	TRUE
Height of rod top z (m)	50	TRUE

  

Inclined Thin Rod 3		Conditions
Gravitational constant G (m <sup>3</sup> /kg s <sup>2</sup> )	6.674E-11	
Density contrast Δρ (kg/m <sup>3</sup> )	500	TRUE
Cross-section A (m <sup>2</sup> )	500000	TRUE
Inclination α (°)	135	TRUE
Length of rod L (m)	2000	TRUE
Height of rod top z (m)	50	TRUE

  

**Range of values for the model:**  
 Δρ - change between 400 and 600  
 L - change between 500 and 2000  
 z - change between 50 and 1700  
 α - change between 10 and 170°  
 All conditions **must** be TRUE!

  

Change values of Δρ, α, L and z in the orange cells to see how the gravity profiles of the rods change. Set different α for each rod to see how the gravitational profile changes with inclination.  
**Note:**  
 Start with the same values of variables (i.e. Δρ = 500; L = 2000; z = 50). Change α for each rod (i.e. α = 45, 90, 135) to see how inclination affects the gravity profiles of the rods. Change values by small increments! Experiment with different values to see the effect on the gravity profiles.

  

First derivative of the function  $g_{z(x)}$   

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 This function represent the instantaneous rate of change of the function  $g_{z(x)}$ . It shows how fast or slow the values change as the point P moves along the horizontal axis. The values are constant for areas of constant slope and show the most variation where slope changes quickly.

  

Second derivative of the function  $g_{z(x)}$   

$$\theta_{z(x)}'' = \frac{d\theta_{z(x)}}{dx} = \frac{\text{change of slope of } \theta_{z(x)}}{\text{unit of length}}$$
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Faulted horizontal sheet A		Conditions
Gravitational constant G (m <sup>3</sup> /kg s <sup>2</sup> )	6.674E-11	
Density contrast Δρ (kg/m <sup>3</sup> )	1000	TRUE
Bed thickness t (m)	100	TRUE
Depth to upthrown block z1 (m)	100	TRUE
Depth to downthrown block z2 (m)	150	TRUE
Fault inclination θ (°)	90	TRUE

  

Gravitational attraction of a faulted horizontal bed		Conditions
Gravitational constant G (m <sup>3</sup> /kg s <sup>2</sup> )	6.674E-11	
Density contrast Δρ (kg/m <sup>3</sup> )	1000	TRUE
Bed thickness t (m)	100	TRUE
Depth to upthrown block z1 (m)	100	TRUE
Depth to downthrown block z2 (m)	150	TRUE
Fault inclination θ (°)	90	TRUE

  

**Reverse fault**  
 When α > 90° then the sheets will overlap and create an area of excess mass. This will show as a high in the gravity profile (i.e. α = 120°).

  

**Range of values for the model:**  
 α - change between 60 and 150  
 z1 - change between 300 and 600  
 z2 - max throw is 50m; z2 is from range (z1, z1+50)  
 All conditions **must** be TRUE!  
 Change values of z1, z2, α in the orange cells to see how the gravity profiles of the faulted bed change. Start with (z1=300, z2=350, α=90). Change values by small increments! Experiment with different values to see the effect on the gravity profile.

  

**Normal fault**  
 When α < 90° then the sheets will gap and create an area of mass deficiency. This will show as a low in the gravity profile (i.e. α = 60°).

  

First derivative of the function  $g_{z(x)}$   

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 This function represents the instantaneous rate of change of the function  $g_{z(x)}$ . It shows how fast or slow the values change as the point P moves along the horizontal axis. The values are constant for areas of constant slope and show the most variation where slope changes quickly.

  

Second derivative of the function  $g_{z(x)}$   

$$\theta_{z(x)}'' = \frac{d\theta_{z(x)}}{dx} = \frac{\text{change of slope of } \theta_{z(x)}}{\text{unit of length}}$$
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Simple two-layer model of propagating seismic phases		Conditions
Offset x (m)	500	
Arrival time of direct phase t <sub>d</sub> (s)	0.278	TRUE
Arrival time of critically refracted phase t <sub>cr</sub> (s)	0.255	TRUE
Time since explosion t <sub>0</sub> (s)	0.299	TRUE
Velocity in layer 1 v <sub>1</sub> (m/s)	1800	TRUE
Thickness of layer 1 z <sub>1</sub> (m)	100	TRUE
Velocity in layer 2 v <sub>2</sub> (m/s)	3000	TRUE
Angle of critical refraction θ <sub>c</sub> (°)	36.87	TRUE

  

**How to use:**  
 1. Change offset in cell B2 (from 0 to 1000) and v1(B7), v2(B8) and z1(B7) to see how long it takes for each phase to arrive (cells B3, B4, B5) from the source to the chosen offset. This is to emphasize the fact that each phase arrives at a different time! Equations for time arrivals are below:  

$$t_d = \frac{x}{v_1}$$

$$t_{cr} = \frac{x}{v_2} + \frac{2z_1 \sqrt{v_2^2 - v_1^2}}{v_2 v_1}$$

$$t_{ref} = \frac{\sqrt{x^2 + 4z_1^2}}{v_1}$$

  

2. The last geo/hydrophone is 1000m away from the source. By setting the offset to 1000 in B2 you can see how long it takes for each phase to arrive to the last geo/hydrophone B3, B4, B5.

  

3. Change the "time since explosion" in B6 (between 0 and the longest arrival time for offset 1000m). As you change this value by small increments (0.1s or less) in graph 1 you can see where the different seismic phases for each geo/hydrophone are at the chosen "time since explosion".

  

4. As the seismic phases arrive to the geo/hydrophones they are recorded as crosses (only in here) in graph 2.

  

5. By connecting the crosses for each phase we get graph 3 which shows travel-time curves of seismic phases. **This, however, is only valid for a simple two-layer model!!!**  
 The direct arrivals blue travel-time curve is a straight line with slope  $m = \frac{1}{v_1}$   
 The critically refracted arrivals red curve is a straight line with slope  $m = \frac{1}{v_2}$   
 The reflected arrivals green curve is a hyperbola.

  

6. Notice where the blue and red lines intersect. This is the cross-over distance where the direct ray is overtaken by the critically refracted ray. Beyond this offset the first arrival is always the critically refracted ray.  
 The reflected ray is never the first arrival.

  

7. Here is a good interactive online module covering seismic acquisition:  
<http://www.learninggeoscience.net/free/00001/index.htm>

## Project timeline

The main work will be undertaken in the summer of 2013 and the new materials are to be ready for use and evaluation with the commencement of the second term in 2013-2014 academic year.