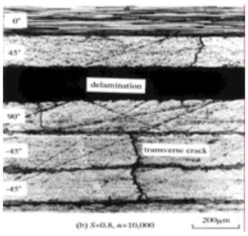
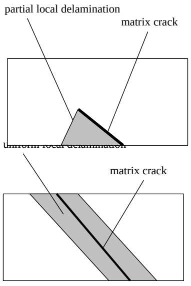
ANALYSIS OF FATIGUE DAMAGE MECHANISMS AND RESIDUAL PROPERTIES OF FIBRE-REINFORCED POLYMER MATRIX COMPOSITE LAMINATES

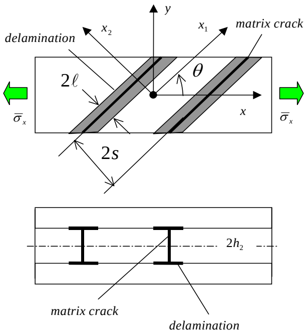
Introduction

Matrix cracking parallel to the fibres is the initial failure mechanism in continuous fibre-reinforced composite laminates under static or fatigue in-plane tensile loading.

It causes degradation of the overall stiffness properties of the laminate and triggers development of other damage modes such as delaminations.







Crack density (crack/cm)

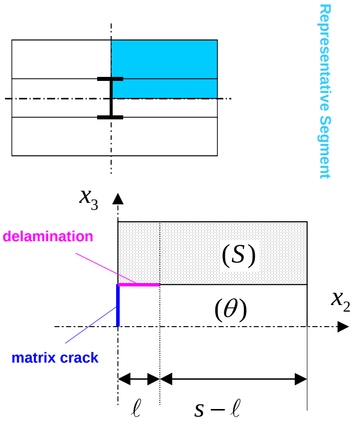
$$C^{mc} = (2s)^{-1}$$

Relative crack density

$$D^{mc} = h_2 / s$$

Relative delamination area

$$D^{ld} = \ell / s$$



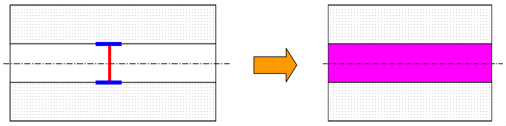
•Angle-Ply Laminate Damaged by Matrix Cracking and Delaminations

Theoretical Modelling

Equivalent Constraint Model

Damaged laminate

Equivalent constraint laminate



Constitutive equations

$$\boxed{\{\bar{\sigma}^{(2)}\} = [\bar{Q}^{(2)}]\{\bar{\varepsilon}^{(2)}\}}$$

$$\bar{\sigma}^{(2)}_{j2} = \left(\sum_{k=1}^{2} A_{kj} \frac{D^{mc}}{\lambda_k h_2} \tanh \frac{\lambda_k h_2 (1-D^{ld})}{D^{mc}} + C_j (1-D^{ld}) \right) \bar{\sigma}_x$$

$$\{\bar{\varepsilon}^{(2)}\} = \{\bar{\varepsilon}^{(1)}\}$$

Residual in-plane stiffness matrix of the ‘equivalent’ layer

Local co-ordinates

$$[\bar{Q}^{(2)}] = [\hat{Q}^{(2)}] - \begin{bmatrix} (\hat{Q}^{(2)}_{12})^2 / \hat{Q}^{(2)}_{22} \Lambda^{(2)}_{22} & \hat{Q}^{(2)}_{12} \Lambda^{(2)}_{22} & 0 \\ \hat{Q}^{(2)}_{12} \Lambda^{(2)}_{22} & \hat{Q}^{(2)}_{22} \Lambda^{(2)}_{22} & 0 \\ 0 & 0 & \hat{Q}^{(2)}_{66} \Lambda^{(2)}_{66} \end{bmatrix}$$

In-situ Damage Effective Functions

$$\boxed{\Lambda^{(2)}_{jj} = \Lambda^{(2)}_{jj}(D^{mc}, D^{ld})}$$

$$\Lambda^{(2)}_{22} = 1 - \frac{\bar{\sigma}^{(2)}_{22}}{\hat{Q}^{(2)}_{12}\bar{\varepsilon}^{(2)}_{11} + \hat{Q}^{(2)}_{22}\bar{\varepsilon}^{(2)}_{22}}, \quad \Lambda^{(2)}_{66} = 1 - \frac{\bar{\sigma}^{(2)}_{12}}{\hat{Q}^{(2)}_{66}\bar{\gamma}^{(2)}_{12}}$$

Global co-ordinates

$$[\bar{Q}]_2 = \mathbf{transformation\_formulae}([\bar{Q}^{(2)}], D^{ld}, D^{mc})$$

Strain Energy Release Rates (SERRs)

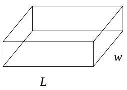
Matrix cracking

$$\boxed{G^{mc} = -\left.\frac{\partial U}{\partial A^{mc}}\right|_{\{\bar{\varepsilon}\}, A^{ld}}}$$

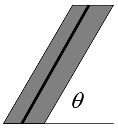
Local delamination

$$\boxed{G^{ld} = -\left.\frac{\partial U}{\partial A^{ld}}\right|_{\{\bar{\varepsilon}\}, A^{mc}}}$$

Total strain energy stored in the laminate



$$U = \frac{1}{2} wL \{\bar{\varepsilon}\}^T \sum_i [\bar{Q}]_i h_i \{\bar{\varepsilon}\}$$



$$\boxed{G^{mc} = -h_2 \{\bar{\varepsilon}\}^{\mathrm{T}} \frac{\partial [\bar{Q}]_2}{\partial D^{mc}} \{\bar{\varepsilon}\} \, |\sin\theta|}$$

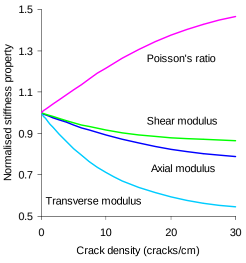
$$\boxed{G^{ld} = -\frac{h_2}{2} \{\bar{\varepsilon}\}^{\mathrm{T}} \frac{\partial [\bar{Q}]_2}{\partial D^{ld}} \{\bar{\varepsilon}\} \, |\sin\theta|}$$

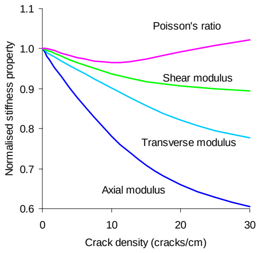
Numerical Results

Matrix Cracking: Stiffness Reduction

Material: glass/epoxy

Lay-ups: $[30/-30]_s$ and $[55/-55]_s$



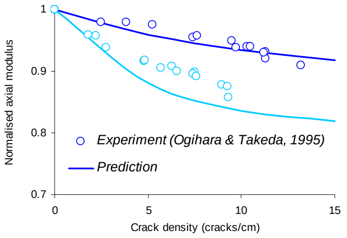


Delamination: Stiffness Reduction

Material: T800/3631 carbon/epoxy

Lay-ups: $[0/90_4]_s$, $[0/90_8]_s$

Relative delamination area: $D^{ld} = 0.1$

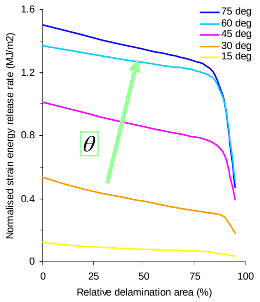


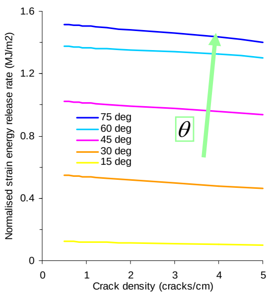
Delamination: Strain Energy Release Rate

Material: AS4/3506-1 graphite/epoxy

Lay-up: $[0_2/\mathrm{theta}_2/\mathrm{-theta}_2]_s$

$$\theta = 15°, 30°, 45°, 60°, 75°$$





Conclusions

•An approach based on the Equivalent Constraint Model and the 2-D shear lag method has been developed and applied to analyse damage mechanisms typically exhibited in angle ply laminates subjected to in-plane tensile loading.

•The approach enabled us to derive closed form expressions for strain energy release rates associated with matrix cracking and uniform local delaminations.

•As opposed to O’Brien’s expression, the present approach gives strain energy release rate for delamination that depends both on matrix crack density and delamination area and takes into account the cumulative effect of damage.

REFERENCES
Kashtalyan M, Soutis C. Stiffness and fracture analysis of laminated composites with off-axis ply matrix cracking. *Composites Part A: Applied Science and Manufacturing* (2007) **38**(4), 1262-1269.
Kashtalyan M, Soutis C. Modelling off-axis ply matrix cracking in continuous fibre-reinforced polymer matrix composite laminates. *Journal of Materials Science* (2006) **41**(20), 6789-6799.
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