Techniques for Inferring Mileage from the Department for Transport's MOT Data Set

R. Eddie Wilson

Jillian Anable (Aberdeen), Sally Cairns (TRL/UCL), Tim Chatterton (UWE), Oliver Turnbull (Bristol) and others

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Faculty of Engineering
University of Bristol

March 25, 2015

UK MOT (Ministry of Transport) test

VT20 MOT Test Certificate VOSA

This certificate has been issued according to the conditions and notes on the back of this certificate.

Vehicle & Operator Services Agency

Note: If you have doubts as to whether this certificate is valid, please use the service described in note 3 overleaf to check.

MOT test number: 761710136293
Make: VAUXHALL
Odometer reading: 105420 Miles
Registration mark: T203UNP
Model: ASTRA
Test class: IV
Vehicle identification or chassis number: WOLOTGF35X8091395
Colour: WHITE
Approximate year of first use: 1999
Expiry date: AUGUST 25th 2007 (ZERO SEVEN)
Issue date/time: AUGUST 18th 2006 (ZERO SIX) 13:30
Fuel type: Petrol
Authentication number
Design gross weight (goods vehicles): kg
Advisory Notice Issued: NO
Test station number: 80572

084907914489518556410227

For all vehicles with more than 8 passenger seats
Seat belt installation checked this test: N/A
Number of seat belts fitted at time of installation check: N/A
Previous installation check date: N/A

Issuer's name in CAPITALS: D. S. BRYANT
Signature of Issuer

Warning: A test certificate is not evidence that the vehicle is in a satisfactory condition.

Check carefully that the above details are correct.
Do not accept a certificate which has been altered.

Reg Mark: T203UNP
Make: VAUXHALL
VTS Number: 80572
MOT Expiry: AUGUST 25th 2007 (ZERO SEVEN)

Peel here

Inspection Authority
HANHAM MOTOR COMPANY
126 BRYANTS HILL
ST GEORGE
BRISTOL
BS5 8RJ

- MOT: the UK's annual safety inspection for all road vehicles older than 3 years
- Since 2005: the results have been captured and stored digitially
- Since November 2010 — the DfT has published this data online - spanning back to 2005.
- Key interest: the *odometer reading* recorded at each test.

A sample of the published data

```
626966|2010-01-18|4|N|P|38198|DE|BMW|523I SE TOURING AUTO|GREEN|P|2494|1998
626977|2010-03-03|4|N|P|25864|ST|LAND ROVER|FREELANDER HSE TD4|BLACK|D|2179|2007
626984|2010-03-04|4|N|P|32884|YO|LAND ROVER|RANGE ROVER SP HSE TDV8 A|BLACK|D|3628|2007
626991|2010-03-26|4|N|F|91196|PL|MERCEDES|ML 320 AUTO|SILVER|P|3199|2000
627020|2010-02-02|4|N|PRS|29180|DH|MERCEDES|ML 320 CDI SE AUTO|SILVER|D|2987|2006
627023|2010-02-24|4|F|P|62713|MK|BMW|325I SE AUTO|SILVER|P|2494|2001
627024|2010-02-24|4|N|F|62713|MK|BMW|325I SE AUTO|SILVER|P|2494|2001
627025|2010-02-22|4|N|F|62647|LU|BMW|325I SE AUTO|SILVER|P|2494|2001
627041|2010-03-04|4|PL|P|230304|IP|MERCEDES|300TE AUTO|GREY|P|2962|1990
627042|2010-03-04|4|N|F|230304|IP|MERCEDES|300TE AUTO|GREY|P|2962|1990
627050|2010-01-25|4|N|PRS|62624|IP|UNCLASSIFIED|UNCLASSIFIED|GREY|P|5300|2006
627058|2010-02-08|4|N|P|88480|SS|JAGUAR|S-TYPE V6 SE AUTO|BLUE|P|2967|1999
627109|2010-01-29|1|N|P|1244|CO|UNCLASSIFIED|UNCLASSIFIED|WHITE|P|125|1959
627145|2010-03-25|7|N|P|35194|LE|AUSTIN|UNCLASSIFIED|BLUE|D|0|1963
627185|2010-02-18|4|PL|P|170507|EX|VOLVO|850|MAROON|P|2435|1997
627186|2010-02-15|4|N|F|170449|EX|VOLVO|850|MAROON|P|2435|1997
627227|2010-02-24|4|N|P|73195|NW|MERCEDES|E430 AVANTGARDE AUTO|BLACK|P|4266|2002
627242|2010-02-01|4|N|P|38225|IP|TOYOTA|HILUX INVINCIBLE D-4D A|BLACK|D|2982|2007
627280|2010-03-08|4|PR|P|44132|B|AUDI|TT QUATTRO (180 BHP)|BLACK|P|1781|2000
627281|2010-03-08|4|N|F|44132|B|AUDI|TT QUATTRO (180 BHP)|BLACK|P|1781|2000
```

- But the tests are grouped by year and do not “link” the vehicles (a problem fixed in more recent releases — at my prompting!)

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```
118173532|2009-08-05|4|N|P|132299|BS|VAUXHALL|ASTRA LS 8V|WHITE|P|1598|1999
118173533|2008-08-11|4|PR|P|123259|BS|VAUXHALL|ASTRA LS 8V|WHITE|P|1598|1999
118173534|2008-08-11|4|N|F|123259|BS|VAUXHALL|ASTRA LS 8V|WHITE|P|1598|1999
118173535|2007-08-13|4|N|P|113709|BS|VAUXHALL|ASTRA LS 8V|WHITE|P|1598|1999
118173536|2006-08-18|4|N|P|105420|BS|VAUXHALL|ASTRA LS 8V|WHITE|P|1598|1999
118173537|2005-08-26|4|N|P|99777|BS|VAUXHALL|ASTRA LS 8V|WHITE|P|1598|1999
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- We can follow individuals around and infer their mileage (rate) between consecutive test dates!!!!

- For example, in the **interval** from 2008-08-11 to 2009-08-05 (359 days), I drove 132,299-123,259 = 9,040* miles, at an **average rate** of 25.18 **miles per day**.

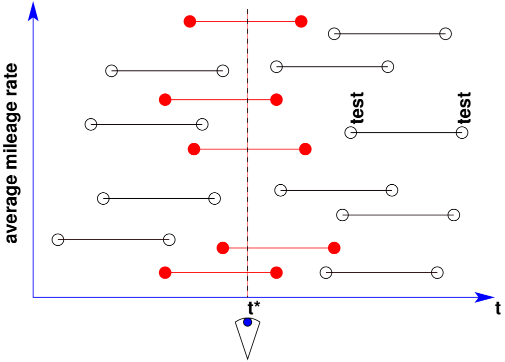
Basic analysis object: *intervals* and their attributes

- Re-arrange blocks of same-vehicle data into consecutive pairs of tests:

Interval	First test			Second test		
	date t_1	miles x_1	place$_1$	date t_2	miles x_2	place$_2$
1	2005-08-26	99777	BS	2006-08-18	105420	BS
2	2006-08-18	105420	BS	2007-08-13	113709	BS
3	2007-08-13	113709	BS	2008-08-11	123259	BS
4	2008-08-11	123259	BS	2008-08-11	123259	BS
5	2008-08-11	123259	BS	2009-08-05	132299	BS

- To which can be linked vehicle-specific attributes:
  VAUXHALL, ASTRA LS 8V, WHITE, P (fuel), 1598 (cc), 1999 (year)
- (Eg) during *interval* 3 — I drove at an average rate of $(123259 - 113709)/364 = 26.24$ miles per day, but we don't know how my mileage was *distributed* during that period.
- These mileage rates are (more or less) complete across the vehicle population — even after cleaning.

Population level statistics: *straddling rate* $\bar{r}(t)$



- Select all N intervals that *straddle* a given *observation date* t^*
- Each interval yields an average (per vehicle) rate r_i.

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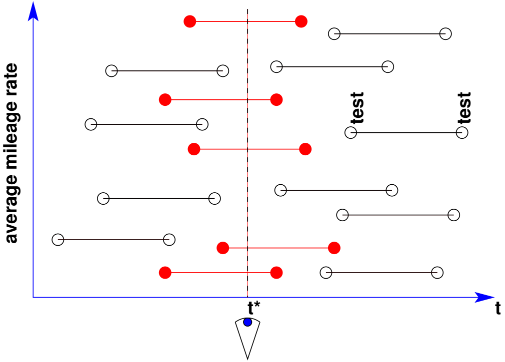

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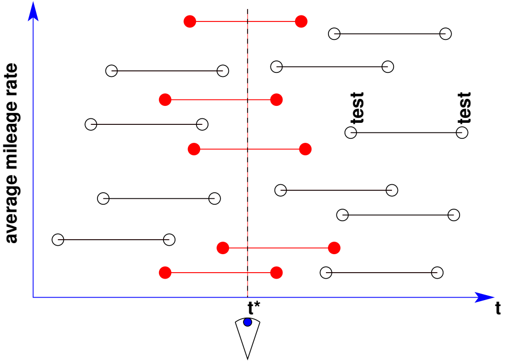

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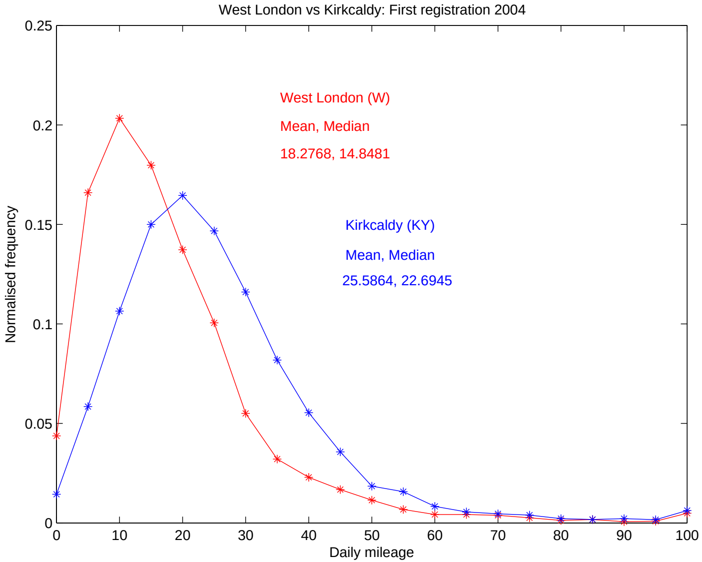
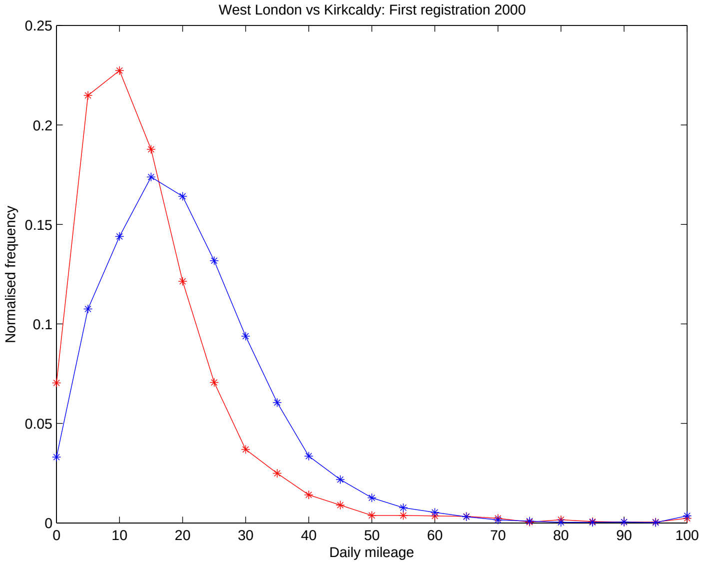
- It is fine for annual statistics: choose $t^* = 1/7/2007$, $1/7/2008$, $1/7/2009$ etc.
- But $\bar{r}(t^*)$ actually incorporates miles driven over the two year span $t^* - 1 \leq t < t^* + 1$.

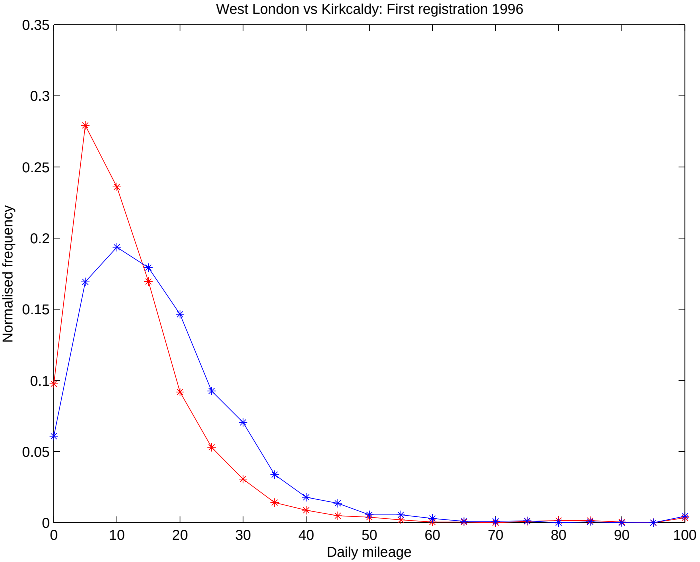
Mileage distributions: new(ish) vehicles



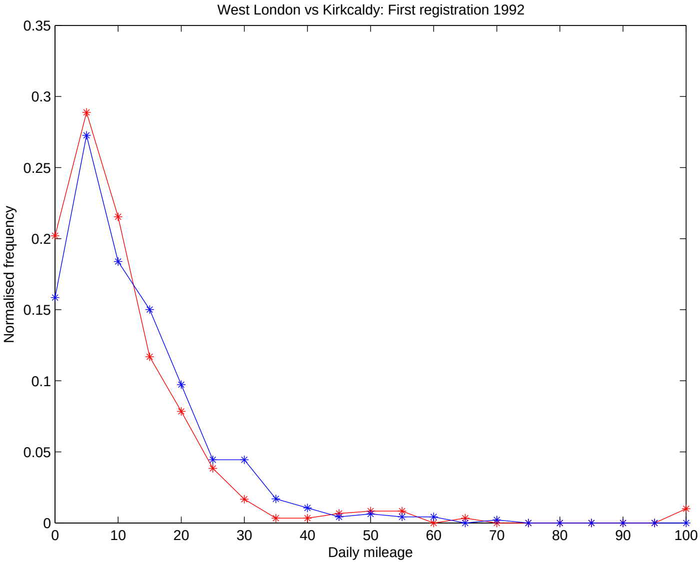
Mileage distributions: older vehicles



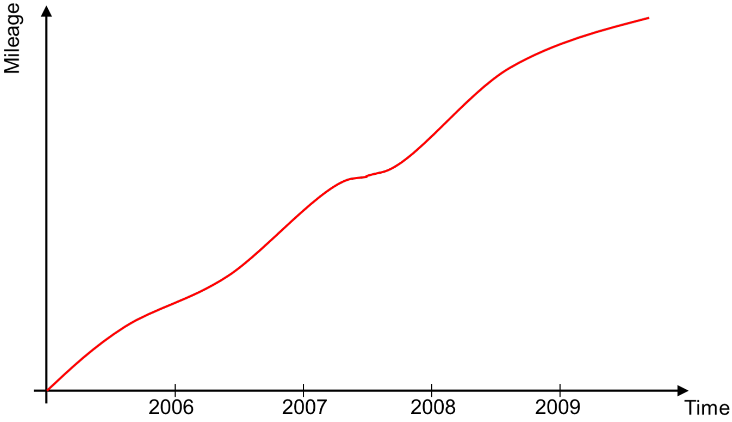
Mileage distributions: even older vehicles



Mileage distributions: old vehicles



From the Straddling Rate to the Census Date Rate



- Progression of a vehicle's odometer with time

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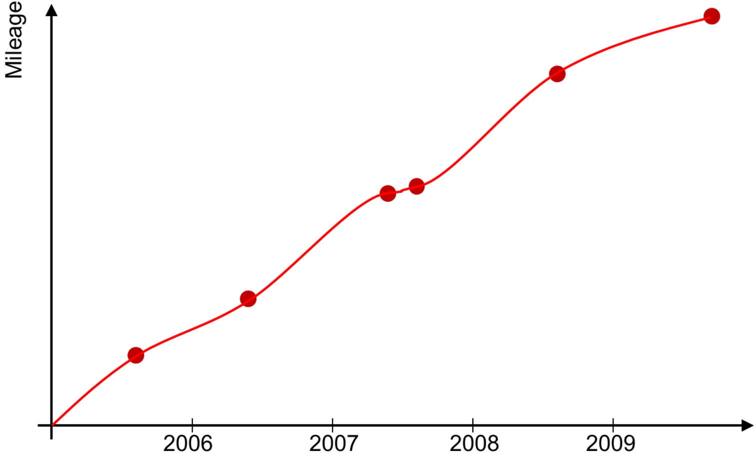

- Progression of a vehicle's odometer with time — with tests

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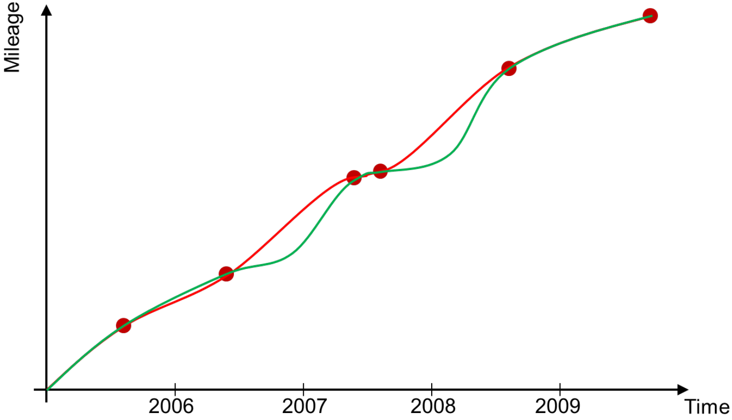

- The tests do not allow you to distinguish the 2 trajectories.

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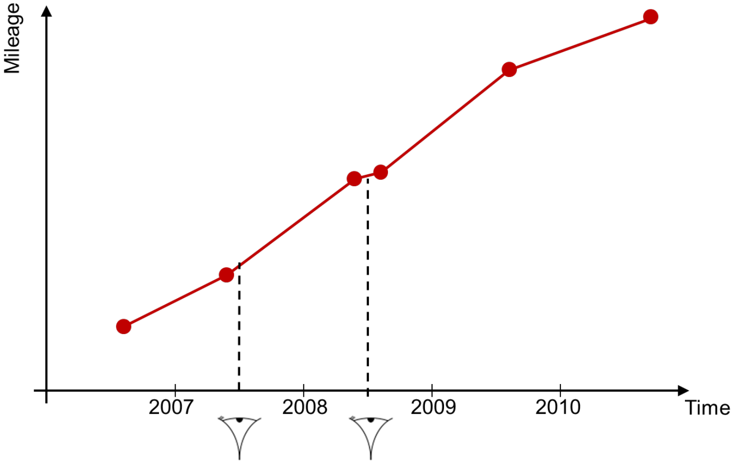

- *Distributions* derived from straddling rate suffer anomalous variance because some intervals are very short

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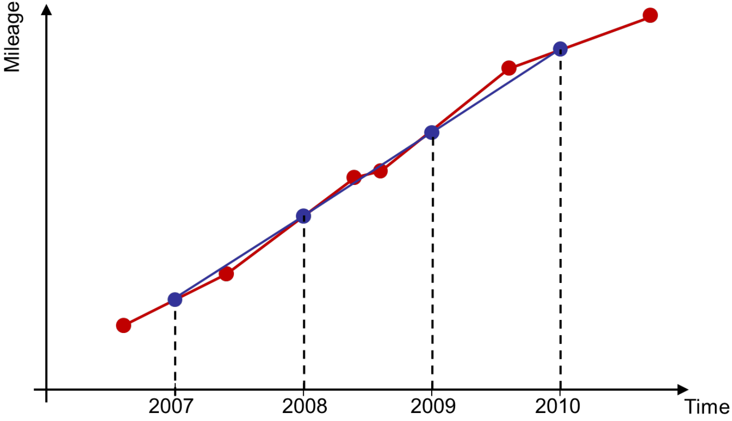

- Solution is to interpolate onto some given *census dates* . . .

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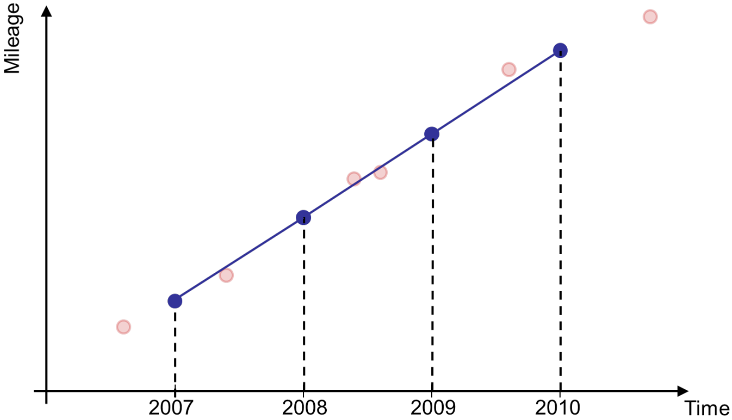
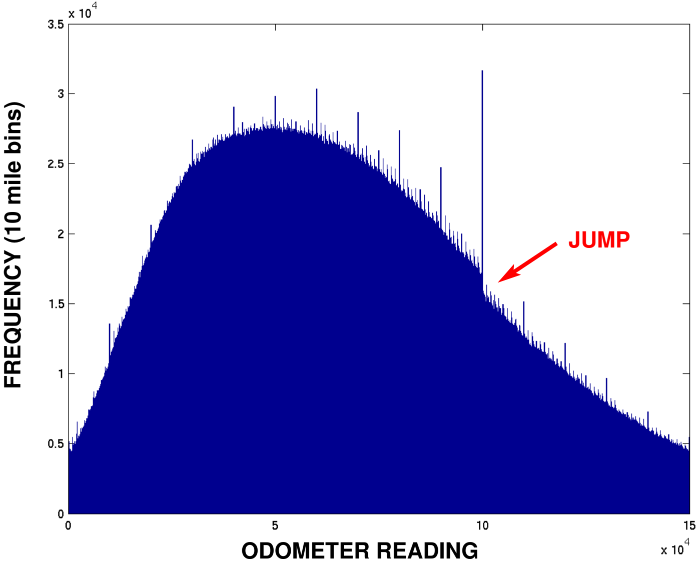


- ... and use the rates between the census dates. (Also neatly synchronises the data into calendar year comparisons.)

Five digit odometer problem



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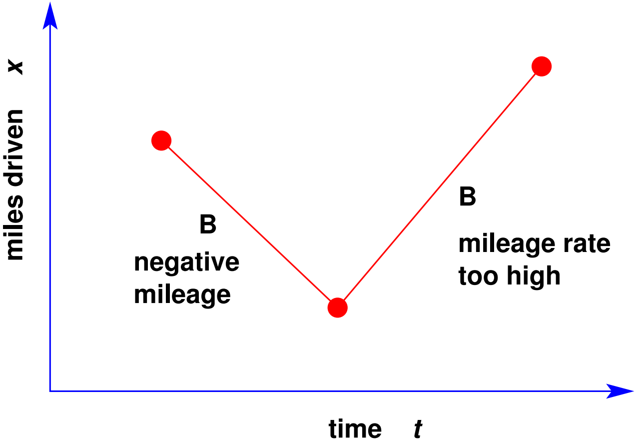
Solution 2: try to identify which individual odometer entries are bad and remove them instead

When two (or more) **B**ads make a **G**ood



- The middle odometer entry is (probably) erroneous — due to a missing digit in the data entry?

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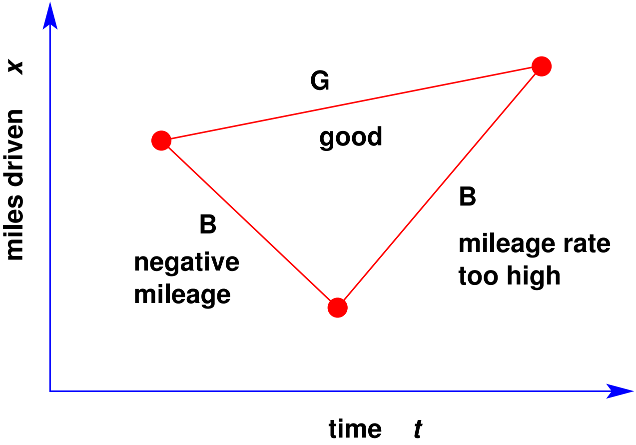

- The middle odometer entry is (probably) erroneous — due to a missing digit?
- The spanning interval without the middle test is (probably) ok.

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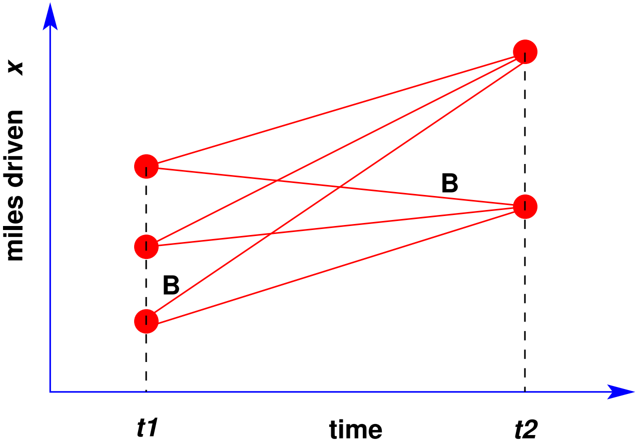
  - Look at removing either or both ends so as to generate **G**. Repeat

How to deal with multiple tests on the same day (I)

(need to pare down to a single odometer reading per test day)



- We want to complete previous syntactic procedure *before* deciding which test to select for each date.

How to deal with multiple tests on the same day (II)

- Compute 4 rates, from the odometer pairs

$$(x_1^{\min}, x_2^{\min}) \quad (x_1^{\max}, x_2^{\max}) \quad (x_1^{\min}, x_2^{\max}) \quad (x_1^{\max}, x_2^{\min})$$

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- Proceed with previous procedure using certainly **B**ad and **G**ood.
- Finally — decide which odometer at each t to use at the end. (For example: the median value.)

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Recall that I cannot possibly say anything about an individual's mileage on finer time scales than one year.

But can I derive something about <u>population level</u> mileage over shorter time scales — eg a month?

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Possible application: detect the sharp drop in driving in Autumn 2008 following Lehman brothers collapse.

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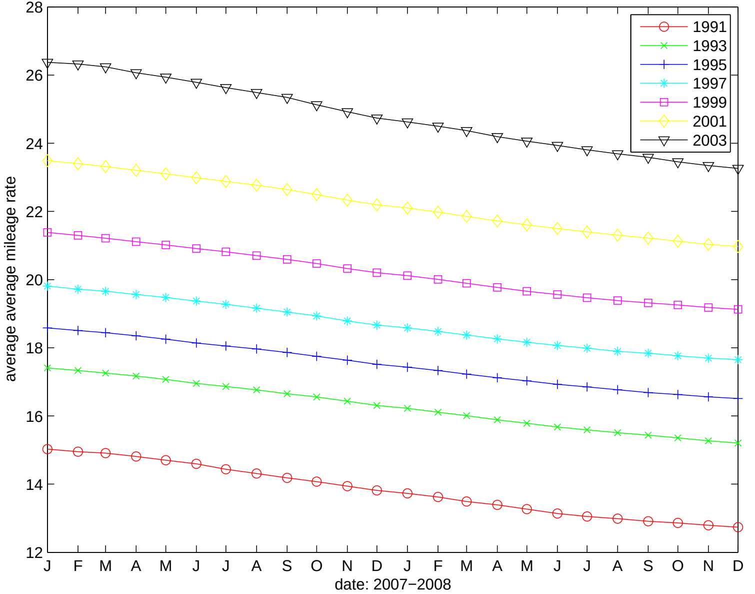
- Compute corresponding $\overline{r}(t_i)$ using straddling procedure
- Pairs $(t_i, \overline{r}(t_i))$ *reconstruct* $\overline{r}(t)$
- Actually … this process is flawed…
  But just look what we can do with it!!!

Example of temporal evolution via straddling (WRONG)



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- Then each vehicle i has an individual spot rate $\phi_i(t)$ with

$$\phi_i(t) = c_i\phi(t) + \text{noise}.$$

Here c_i=const.; $\langle c_i \rangle = 1$; and $\langle \text{noise} \rangle = 0$, so that $\phi = \langle \phi_i \rangle$.

Basic postulate: the population *spot rate* $\phi(t)$

- Suppose there is a population-level *spot rate* $\phi(t)$ that modulates *all* vehicles' mileage (alt. restrict to a population segment).

- Then each vehicle i has an individual spot rate $\phi_i(t)$ with

$$\phi_i(t) = c_i \phi(t) + \mathrm{noise}.$$

Here c_i=const.; $\langle c_i \rangle = 1$; and $\langle \mathrm{noise} \rangle = 0$, so that $\phi = \langle \phi_i \rangle$.

- Let $\psi_i(\tau)$ denote miles driven by i between tests at times $\tau - 1/2$ and $\tau + 1/2$. Then

$$\psi_i(\tau) = \int_{\tau-1/2}^{\tau+1/2} (c_i \phi(s) + \mathrm{noise})\, \mathrm{d}s, \qquad = c_i \int_{\tau-1/2}^{\tau+1/2} \phi(s)\, \mathrm{d}s.$$

From the *spot rate* to the *straddling rate*

- Thus by averaging over tests that straddle t:

$$\overline{r}(t) = \int_{t-1/2}^{t+1/2} \langle \psi_i(\tau) \rangle_i \, \mathrm{d}\tau = \int_{t-1/2}^{t+1/2} \langle c_i \rangle \int_{\tau-1/2}^{\tau+1/2} \phi(s) \, \mathrm{d}s \, \mathrm{d}\tau.$$

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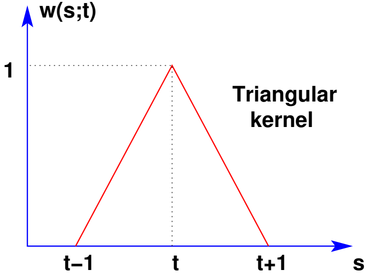
- Simplify integral by $\langle c_i \rangle = 1$ and reverse the order of integration

$$\bar{r}(t) = \int_{t-1}^{t+1} w(s;t) \phi(s) \, \mathrm{d}s,$$



- Thus $\phi(t)$ leads to $\bar{r}(t)$.
  But we want to derive $\phi(t)$ from $\bar{r}(t)$ (which is derivable from data).

From the *straddling rate* to the *spot rate*

- See TR-E 2013 for a whole bunch of Mathematics!!! - upshot:

$$\bar{r}''(t) = \phi(t+1) - 2\phi(t) + \phi(t-1).$$

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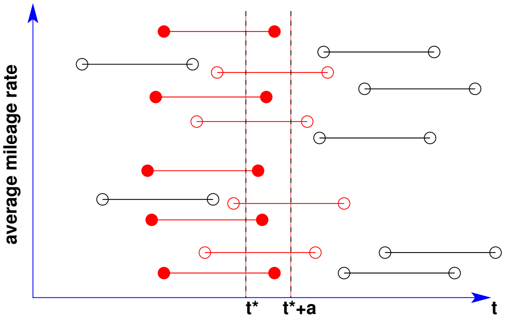
  - in practice: $\overline{r}(t)$ is noisy, so the difference is applied to a smoothing least squares fit spline.
- Unfortunately: 2 years of initial data for $\phi(t)$ are required — at the fine scale resolution Δt.

Refinement of the *straddling rate* idea



- Select only the intervals that *straddle* t^* and with right hand ends before $t^* + \alpha$, with $\alpha \leq 1$ year.
- Call resulting average average *straddle rate* $\bar{r}_\alpha(t)$

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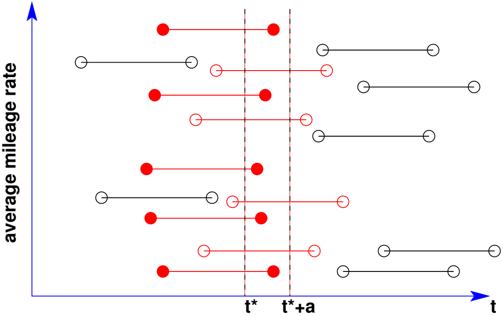

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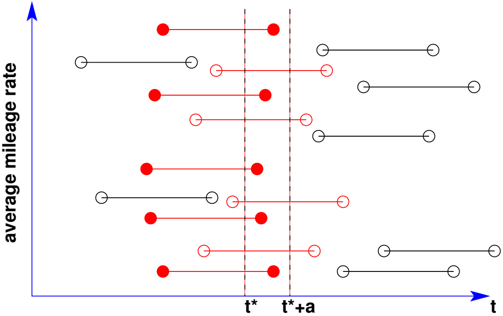

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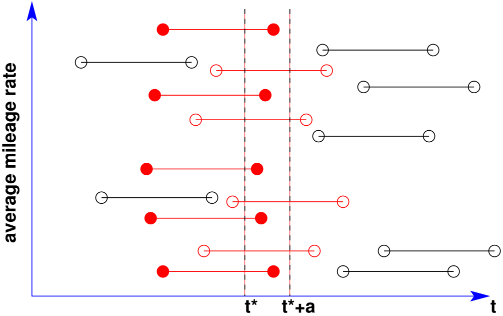

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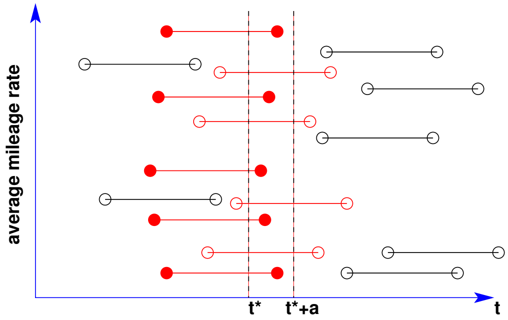

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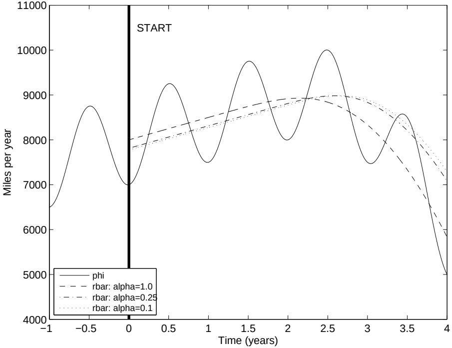
- So interest is in $\alpha \to 0$, which gives $\overline{r}'_\alpha(t) \simeq \phi'(t) - \phi'(t-1)$ (natural meaning)
- $\alpha \to 0$ means fewer and fewer intervals, means noisy $\overline{r}_\alpha(t)$

Synthetic data set-up

- Choose *spot rate*

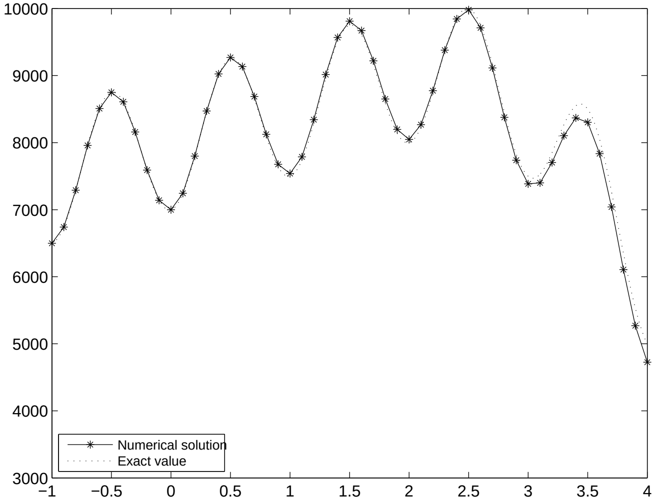
$$\phi(t) = 8000 + 500t - 1000\cos 2\pi t - 1000\left[t-2\right]_+(t-2)^2,$$

- 10^6 vehicles with tests 1 year apart, test dates uniformly distributed through calendar year
- Vehicle i daily mileage drawn from a distribution modulated by $\phi(t)$ and (random) c_i.
- Odometer readings on test dates are synthesised by adding individual vehicle daily totals



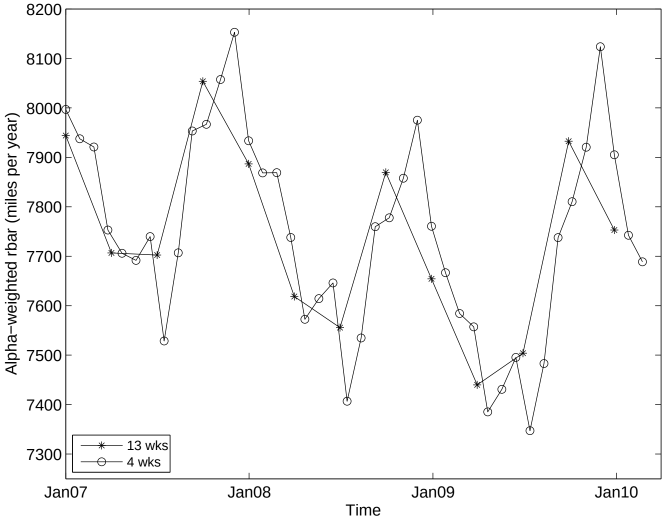
- Periodic component in spot rate $\phi(t)$ is suppressed in straddling rates $\bar{r}_\alpha(t)$

Results with synthetic data: $\alpha = \Delta t = 0.1$ years



- Reconstructed $\phi(t)$ almost indistinguishable from ground truth.

Straddling rates $\bar{r}_\alpha(t)$ for real-world data



- Seasonal component shouldn't be there: underlying assumptions of the theory are broken

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On **A3**: fails because a pattern in new vehicle registrations throughout the year (in the UK).

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- Other data sources might enable huge extensions:
  1. Per vehicle emissions data
  2. Fine scale data (month?) for point of first use
  3. Fine scale location data (LLSOA of registered keepers?)
  4. Link vehicles with same registered keeper / address

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- UK MOT data set: some fixes/patches to theory are needed.
- Please contact me if you know of other datasets (international) in which odometer readings are systematically collected.
- These methods have the potential to complement / replace existing survey-based / link-flow techniques for estimating population-level mileage.