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Long-Term Option Contracts for Carbon Emissions

Professor Alexander G. Kemp and
Professor Joseph Swierzbinski

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Research in North Sea Economics has been conducted in the Economics Department since 1973. The present and likely future effects of oil and gas developments on the Scottish economy formed the subject of a long term study undertaken for the Scottish Office. The final report of this study, The Economic Impact of North Sea Oil on Scotland, was published by HMSO in 1978. In more recent years further work has been done on the impact of oil on local economies and on the barriers to entry and characteristics of the supply companies in the offshore oil industry.

The second and longer lasting theme of research has been an analysis of licensing and fiscal regimes applied to petroleum exploitation. Work in this field was initially financed by a major firm of accountants, by British Petroleum, and subsequently by the Shell Grants Committee. Much of this work has involved analysis of fiscal systems in other oil producing countries including Australia, Canada, the United States, Indonesia, Egypt, Nigeria and Malaysia. Because of the continuing interest in the UK fiscal system many papers have been produced on the effects of this regime.

From 1985 to 1987 the Economic and Social Science Research Council financed research on the relationship between oil companies and Governments in the UK, Norway, Denmark and The Netherlands. A main part of this work involved the construction of Monte Carlo simulation models which have been employed to measure the extents to which fiscal systems share in exploration and development risks.

Over the last few years the research has examined the many evolving economic issues generally relating to petroleum investment and related fiscal and regulatory matters. Subjects researched include the economics of incremental investments in mature oil fields, economic aspects of the CRINE initiative, economics of gas developments and contracts in the new market situation, economic and tax aspects of tariffing, economics of infrastructure cost sharing, the effects of comparative petroleum fiscal systems on incentives to develop fields and undertake new exploration, the oil price responsiveness of the UK petroleum tax system, and the economics of decommissioning, mothballing and re-use of facilities. This work has been financed by a group of oil companies and Scottish Enterprise, Energy. The work on CO2 Capture, EOR and storage is also financed by a grant from the National Environmental Research Council (NERC).

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Long-term Option Contracts for Carbon Emissions

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Professor Joseph Swierzbinksi

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Long-term Option Contracts for Carbon Emissions

Professor Alex Kemp
and
Professor Joseph Swierzbinski[1]

Abstract

This paper proposes and considers the sale of long-term put options for carbon by the UK Government as an element of a programme for encouraging long-term investments to reduce carbon emissions (including carbon capture and enhanced oil recovery/storage schemes).

1. Introduction

The UK Government is firmly committed to achieving dramatic long-term reductions in CO_2 and other greenhouse gas emissions and has already introduced several policy measures to this end. But much remains to be done if the ambitious targets are to be met, and it is generally acknowledged that further policy initiatives will be required. In the recent major review it is emphasised that "a key role for Government is to put in place a framework which, by placing a value on carbon, provides a financial incentive for businesses and households to incorporate the climate change impact of their activities. A carbon price is essential for making lower carbon emissions a business imperative".[2]

Current UK policies include several elements which either directly produce a value for carbon, as with the Climate Change Levy (CCL), or do so indirectly, as with the European Union Emissions Trading Scheme (EU ETS), and the Renewables Obligation.[3] The total CCL package includes not only the Levy on the business use of energy (introduced in 2001), but Climate Change

1 Professor Swierzbinski wishes to acknowledge financial support for his research from the UK Environment Agency.
2 See DTI, (July, 2006) p.27.
3 There are other forms of incentives including enhanced capital allowances for some long life plant and machinery including good quality CHP assets.

Agreements (CCAs) which at March 2006 covered 42 energy intensive sections of industry and around 10,000 facilities. The CCAs permit an 80% discount from the CCL when agreements to meet energy efficiency targets have been reached with the Department of Environment, Food and Rural Affairs (DEFRA). The present agreements last until 31st March, 2013.[4] In the draft Climate Change Bill published on 13th March, 2007 it is proposed that a system of five-year carbon budgets be set 15 years ahead in order to give more certainty to investors. It is noteworthy that in the draft Bill the Government foresees the possibility that it may itself engage in the purchase of (overseas) emissions credits and borrow emissions rights on an intertemporal basis.

The UK Government has clearly indicated its commitment to the EU ETS as the "best long-term mechanism for securing least cost emissions reductions [and] it will remain the central element of the UK's emissions reductions policy framework".[5] The short experience of the scheme to date has highlighted teething problems with the details of the allocation of the CO_2 allowances. The result has been that their value has been quite volatile from a high exceeding €30 per tonne in April 2006 to little more that €1.2 per tonne in February/March 2007. In the latter period the price for delivery of CO_2 allowances in 2008 has been €14-€15 per tonne.

There is general agreement that to encourage long-term investment in carbon-abatement activities there needs to be more certainty on emissions allocations which have an obvious effect on the market value of the related allowances. Currently there is so much uncertainty surrounding the prospective value of allowances that they almost certainly cannot be usefully employed in making long-term investment decisions. Against this background there is a case for

4 See H M Treasury (March, 2006) for a fuller description.
5 See DTI, (July, 2006) p. 30.

intervention by the Government (or its agent) to reduce this uncertainty. In this paper a scheme which can achieve this is proposed and discussed. The scheme is consistent with the Government's objectives as outlined above. It is stressed that the proposal is only one element in what might be a package of measures including tax and other capital incentives.

2. The Proposed Scheme in Outline

Under the proposed scheme the Government (or its agent) would sell long-term put options for carbon emissions. These would involve the commitment by the Government to purchase from the owners of the options a specified amount of carbon allowances at a fixed price at some future date. It is suggested that the scheme could constitute a low cost, feasible way of reducing the long-term price uncertainty currently faced by investors in carbon abatement schemes. Specific advantages are summarised as follows:

a) An option represents a contract between the Government and the owner of the option that resembles a Government bond. Hence, the sale of put options for carbon provides a mechanism for the Government to credibly commit to a minimum future price for those carbon emissions covered by the option.

b) The Government can raise revenue in the present by the sale of such options.

c) The ownership of the options by firms provides a hedge against the future price risk.

d) When the price of carbon emissions in the future is sufficiently high, the future cost to the Government incurred by selling a put option for carbon emissions is zero. Moreover, it is easy to calculate an upper bound on the expected cost to the Government of selling such options.

e) if it is so desired, the properties of these options can be tailored to meet the needs of specific projects.

f) If it is so desired, put options for carbon could be transferable and, hence, tradeable. As in the case of the UK and EU markets for carbon emissions, the issue of tradeable put options for carbon by the UK Government would be an example of the use of a market mechanism to encourage cost-effective reductions in carbon emissions. As the EU market for carbon continues to evolve, options on future allowances traded on the EU market might be developed that had similar characteristics (and, hence, similar costs and values) to the put options discussed in this paper.

g) A well-developed market for carbon emissions is not necessary for the put options discussed in this paper to be a useful policy instrument. Nor is it necessary that the put options be transferable. The price that a firm pays to the Government for put options associated with a particular project could be determined by a bilateral negotiation between the firm and the Government. In addition to the price, other features of the option such as the level of the price floor embodied in the option and the time to expiration could also be subject to negotiation.

h) Because of the long-term nature of many investments for reducing carbon emissions, the options considered in this paper may also be of long duration (but not as long as the life of a typical project).

3. Options Widely Employed in Business Decisions Including Relations with Governments

Options are currently used in finance and business in a variety of ways. The volume of trade in short term financial options is enormous. Longer term options are an important feature of many business investments. In the oil industry, for example, it is well understood that an offshore oil lease or licence issued by a host Government can be usefully thought of as a long-term call option to explore for and develop oil reserves.[6] The UK Government has issued such long-term licences since 1964. In a text intended, in part, for businessmen, Copeland and Antikarov (2003) provide numerous examples of investment projects that include options on real assets (so-called "real options"). Cox and Rubinstein (1985) discuss markets for financial options as well as presenting a detailed discussion of the famous Black-Scholes formula for valuing a call option. Dixit and Pindyck (1994 and 1995) provide a detailed analysis of the use of options in long-term business investment decision-making.

4. Long-Term Put Options for Carbon

In order to introduce the main ideas using a simple concrete example, this section focuses on the properties of the simplest type of put option, a European put option.

A European put option provides the owner of the option with the right to sell a specified asset at a specified price (the exercise price) on a specified date (the expiration date).

[6] In an influential paper, Paddock, Siegel, and Smith (1988) showed how an approach based on options could be used to estimate the value of an offshore oil lease.

The original seller of the option, sometimes called the writer of the option assumes the obligation to purchase the asset if the owner of the option chooses to exercise his option. The exercise price of the option is commonly denoted by *K*. The number of time periods between the present and the expiration date is often denoted by *T*.

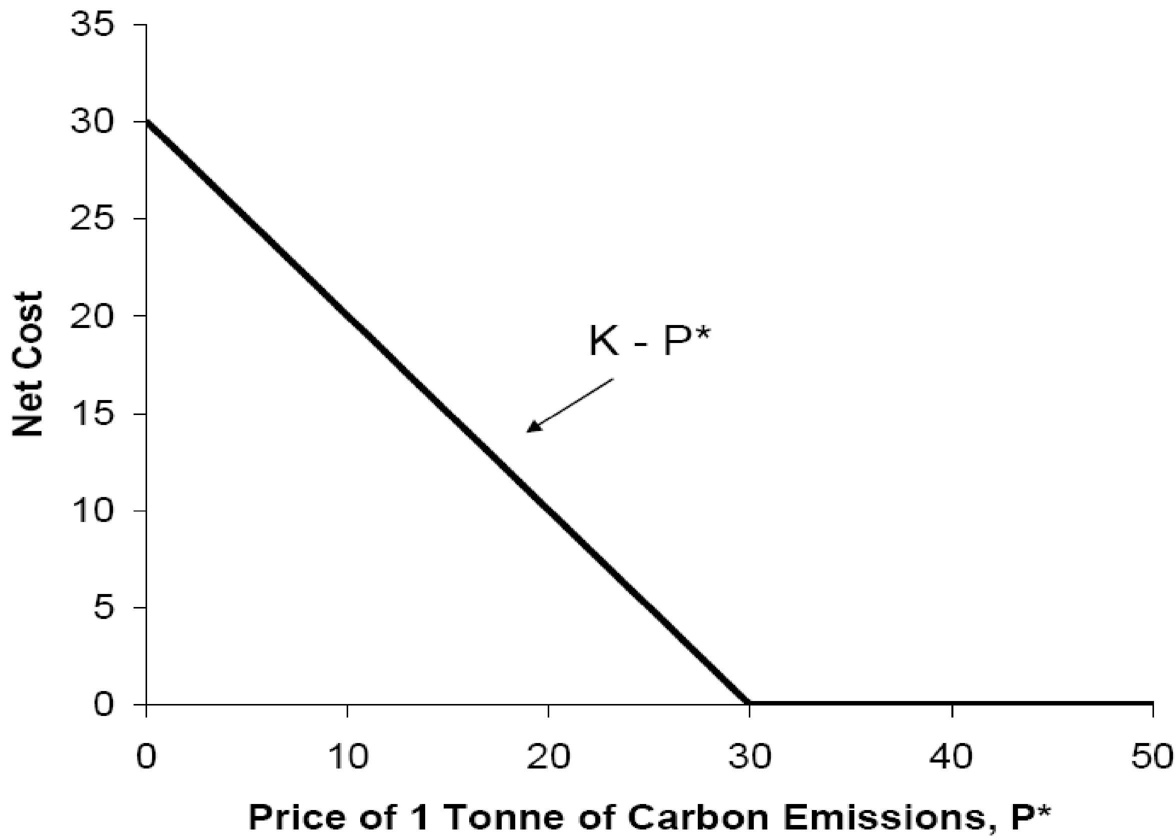
For example, a European put option for 1 tonne of carbon with an expiration date of 31 January, 2020 and an exercise price of €30 per tonne would give the owner of the option the right to sell to the writer of the option 1 tonne of carbon emissions on 31 January, 2020 for a price of € 30.

Ownership of a put option differs from the sale of a futures contract because the owner of an option is not obligated to exercise the option. Since the exercise of a put option is voluntary, the owner of the option will only exercise the option when it is in his best interest to do so.

If there were no market for carbon emissions, then the owner of a put option who wished to sell reductions in carbon emissions would always exercise the option. When a market for carbon emissions exists, then the owner must decide whether to sell in the market or to exercise the option. If the market price for carbon at the expiration date were less than the exercise price of the option, then the owner of the put option should exercise the option and sell at the exercise price. If, on the other hand, the market price were greater than the exercise price, then it would be more profitable to sell carbon emissions in the market and let the option expire unused.

Figure 1 shows how the net cost to the seller of a put option depends on the market price of carbon emissions when the exercise price of the option is € 30 per tonne.

Figure 1. Net Cost Incurred by the Seller of a European Put Option at the Expiration Date (Exercise Price = € 30)



Let P^* denote the market price of 1 tonne of carbon emissions at the date at which a put option with exercise price K expires. (P^* is the price, at the expiration date of the option, of an emission allowance that permits the holder of the allowance to emit 1 tonne of carbon.) When $P^* < K$, the owner of the option can be expected to exercise it, and the seller of the option will have to pay out K for 1 tonne of carbon. However, the seller can partially offset this expenditure by reselling the carbon purchased in the market for P^* so that the net expenditure or net cost to the seller of the option is only $K – P^*$.[7] When $P^* > K$, the owner of the put option will prefer to let it expire unused, so the cost to the seller is 0.

If X^* denotes the net expenditure or net cost incurred by the seller of the option at the expiration date, then the above discussion can be summarized compactly in terms of the following formula:

$$X^* = \max[K - P^*, 0]. \tag{1}$$

Figure 1 and equation (1) indicate that the maximum cost which the seller of a European put option can incur at the expiration date of the option is the payment of the exercise price, K. Moreover, if the market price of carbon at the expiration date is sufficiently high, then the seller of the option incurs no cost from selling the option.

7 Alternately, the seller may prefer to reduce the overall level of carbon emitted by not reselling the carbon emission reduction. In this case, the seller pays a premium over the market price of $K – P^*$ for buying carbon via the option rather than in the market, so the net cost of the option to the seller is the same.

Figure 2. Revenue Obtained by the Sale of 1 Tonne of Carbon with and without a Put Option (Exercise Price = € 30)

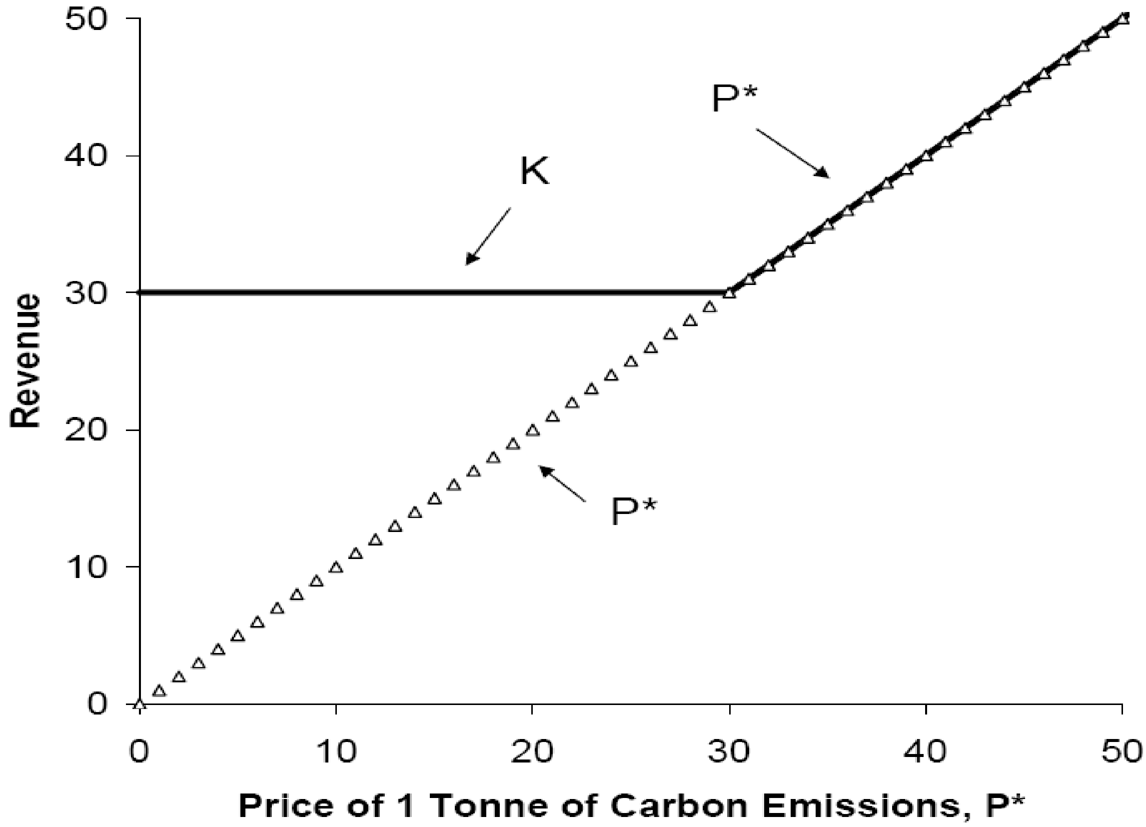


Figure 2 indicates why the ownership of a put option can benefit an investor who expects to sell reductions in carbon emissions at the expiration date of the option. Figure 2 shows the revenue obtained by the sale of 1 tonne of carbon with and without a put option. As in Figure 1, it is assumed that the exercise price of the option is $K = €\,30$.

Without the put option, the revenue obtained by selling 1 tonne of carbon is simply the market price, P^*. This revenue is indicated in Figure 2 by the straight line of small triangles. The revenue obtained when the investor also owns the put option is shown by the heavy solid line in Figure 2. With the put option, the investor exercises the option and obtains the revenue K when $P^* < K$.

When $P^* > K$, the investor sells the carbon in the market and obtains the revenue P^*. Hence, the ownership of a put option provides a prospective seller of carbon with a guarantee that the sales price faced by the seller will never be below the exercise price of the option. Put options provide a mechanism for the seller of the option to provide a credible guarantee of a minimum price for the underlying asset (e.g. carbon emissions) to the owner of the option.

Note that, at the expiration date of the put option, the value of a portfolio consisting of 1 tonne of carbon and 1 put option is simply the sum, $X^* + P^*$.[8]

What is the value in the present of a put option for carbon to investors who expect to be able to sell reductions in carbon emissions at some future date? This value should consist of two components. The first is the difference in the expected present value of the revenue obtained with and without the option. The observation in the previous paragraph implies that this difference is simply the expected present value of the net expenditure, X^*, associated with the option. Let EPV(X^*) denote this expected present value. The second component of value produced by the option is the risk premium which the investor is willing to pay in the present for the reduction in future risk associated with the minimum price guarantee provided by the put option. Let RP denote this risk premium. If V_0 denotes the value to investors in the present of the put option, then the discussion in this paragraph can be summarized by the following equation.

$$V_0 = \mathrm{EPV}(X^*) + \mathrm{RP} \tag{2}$$

8 If $P^* < K$, $X^* + P^* = K - P^* + P^* = K$. If $P^* > K$, $X^* + P^* = P^*$.

The first term on the right-hand side of the above equation, EPV (X^*), depends on (i) the characteristics of the put option, notably K and T, (ii) the riskless interest rate, r, and (iii) the assumptions made about the probability that the future price of carbon emissions, P^*, which is uncertain from the perspective of the present, will take on particular values. The second term on the right-hand side of equation (2), RP, also depends on these factors. In addition, the risk premium may depend on the investors' attitudes toward risk and how the risk associated with the future sale of carbon interacts with other risks in the portfolio of investments held by the investors.

The appendix contains a brief discussion of how EPV(X^*) could be estimated. Table A.1 in the appendix reports some example values for EPV(X^*) for selected values of the various parameters.

The cost in the present to the Government or another seller of put options for carbon should also be given by an equation like equation (2). However, the risk premium for the option seller is the payment which the seller requires to be willing to bear the risk associated with the uncertain future expenditure, X^*, to which the writer of the option commits by selling the option.

If the value to an investor in the present from owning a put option and the cost to the Government in the present of selling a put option are each given by a formula like that in equation (2), what scope is there for a mutually beneficial agreement? There are at least three reasons why such agreements may be feasible. First, the Government's risk premium may be less than that of investors since Government expenditure is spread over a large tax base and a wide set of risks. Indeed, some economists have argued that the Government should be risk neutral with respect to most risks.[9] More generally, it could be argued that Government decision makers should be less risk averse than the shareholders or managers of a particular firm.

A second reason why investors' willingness to pay for put options may be greater than the Government's cost, is that the Government may hold more optimistic assumptions than investors about the future development of markets for carbon emissions. For example, Figure 1 shows that, all other things equal, the expected present value of the future expenditure X^* will be lower when the future price of carbon P^* is expected to be higher.

A final reason why the Government may be willing to consider the sale of put options for carbon are the social benefits that may be produced if the sale of these options stimulates investment in projects to reduce carbon emissions. To the extent that the social benefits from the sale of the option represent an external benefit to present or future citizens, the costs to the Government indicated in equation (2) should, in principle, be reduced by the marginal external social benefit produced by the option. However, in practice it may be very difficult to quantify this marginal benefit.

It is straightforward to calculate a simple upper bound on the present value of the cost to the Government from selling a European put option for 1 tonne of carbon emissions. The worst case scenario from the Government's point of view is that it will always be required to purchase the emissions at time T for exercise price K. In this worst case scenario, there is no risk, so that the cost K at time T should be discounted to the present using the riskless rate of interest, r. Hence,

[9] Arrow and Lind (1970) argue, for example, that the Government should act as if it were risk neutral when evaluating public investments.

the present value of the Government's expenditure in the “worst case” scenario, C_0^{MAX}, is given by the following equation.

$$C_0^{\text{MAX}} = \frac{K}{(1+r)^T} \tag{3}$$

If the value which an investor places on a put option is greater than C_0^{MAX} then there is clearly scope for a mutually beneficial agreement between the investor and the Government.

To illustrate the main idea, this section has focused on the simple case of a single European put option for carbon emissions. Many more complicated options and portfolios of options could be offered for sale. For example, an option could provide the right to sell an asset at a specified exercise price at any time between a specified starting date and a specified expiration date. A put option that can be exercised at any time on or before a given expiration date is called an American put option. An American put option represents the special case where the “starting date” for exercise is the date on which the option is purchased. The value of an American put option will be at least as great as the value of a European option. This follows from the greater flexibility which an American option gives to the owner regarding the timing of the exercise of the option. From the Government’s perspective as writer of the option, this flexibility has some disadvantages as there is uncertainty about the timing of its financial commitment. With the European option the Government knows in advance when its financial commitment may be required. The uncertainty regarding whether the owner will exercise his option still remains, however.

5. Some Potential Advantages of the Put Option Approach

The sale of a put option contract ought to represent a credible commitment on the part of the Government. It is true that policy announcements by Governments regarding, for example, long-term targets for carbon reductions are sometimes greeted with scepticism. However, one could reasonably argue that the sale of a specific contract to purchase future carbon emissions represents a commitment in a way that policy announcements do not. Presumably an option contract could be enforced in court, although the precise extent to which courts can enforce such contracts with the Government is a matter for legal analysis outside the scope of the present paper.

The most effective mechanism for enforcing option contracts sold by Government is probably the effect that a default on such a contract could have on the Government's reputation for honouring its financial commitments. In particular, the market for Government bonds is an extremely important market in many countries including the UK and USA. The bond market is used to raise revenue for the Government and implement macroeconomic policy. Even a very small risk of damaging the Government's credibility in the bond market seems likely to deter defaults on other types of Government contract.

Long-term put options issued by the Government directly address the problem of the long-term risk associated with the future price of carbon emissions. Even if the Government were to commit to a long-term climate change policy, investors would still face the difficult problem of translating such a commitment into an accurate forecast of future prices for carbon emissions. The sale of put options avoids this problem by providing investors with a guaranteed minimum future price for carbon emissions that reduces the investors' risk.

The sale of put options for carbon appears to be a relatively low cost policy for the Government. Other than the transactions cost involved in issuing the option, the sale of put options involves an immediate benefit rather than a current cost. In the present, the Government receives revenue and encourages investments intended to mitigate climate change. In the future, the cost to the Government depends on the future market for carbon emissions and the level of the minimum price for carbon embodied in the option. If the future market price is sufficiently high, there is no future cost to the Government. More generally, it is easy to estimate an upper bound on the present value of the future cost that the Government incurs from the sale of a European put option.

Although the discussion in this paper has focused on the example of a single European put option, the characteristics of options offered for sale can be tailored to meet the needs of investors in a particular project. For example, a portfolio of options with different expiration dates could be sold. Moreover, as has already been observed, different degrees of flexibility can be incorporated into options. Cox and Rubinstein's (1985) book illustrates some of the many patterns of payoffs that can be produced by combinations of options.

6. Some Remaining Questions

As with any contract involving the sale or purchase of reductions in carbon emissions, an important issue that must be resolved for the sale of put options to be an effective policy instrument is the determination of the baseline against which reductions in future carbon emissions are measured. For some projects, the determination of a baseline may be straightforward. For other types of investment, given their novelty, the determination of an appropriate baseline may be more difficult.

An interesting question is the extent to which put options for carbon issued by the Government should be transferable and, hence, potentially tradeable. There appear to be both advantages and disadvantages to the Government from allowing transferable options. Thus a market for transferable put options could provide valuable information about the cost of future emission reductions and provide prospective investors with a low cost means of hedging long-term price risks. However, if the development of such a market is to be encouraged, then the problem of standardizing the characteristics of put option contracts must be addressed. Standardisation is important for reducing the transactions costs involved when a contract is transferred from one party to another. One potentially thorny issue is the problem of creating a standardized baseline for assessing emission reductions.

From the perspective of a particular project, allowing put options to be transferable also appears to offer both advantages and disadvantages to the Government. In particular, the transferability of put options would presumably increase the scrap value of the project for which they were originally purchased. On the one hand, an increase in the scrap value of a project makes it more likely that the project will be cancelled prior to completion. On the other hand, an increase in the scrap value of a prospective project also makes it more likely that the project will be initiated, since the scrap value provides a partial hedge against the risk that completion of the project will become uneconomical in the future. Projects to reduce carbon emissions are likely to involve social benefits that are not fully captured by the investors in such projects. Because of these external benefits, Governments will generally wish to encourage both the initiation and the completion of such projects.

Financial options are generally of short-term duration,[10] though as noted above in Section 3, the concept is employed in very long-term contractual relationships between the Government and business (such as with North Sea oil licences). Put options of the type discussed in this paper with a time to expiration of, say, 10 or 15 years would be rather novel. But given the underlying scope for such contracts as discussed in Section 4 above there is no fundamental reason why they could not be negotiated.

The fundamental question of Government involvement in such transactions also arises. The argument of this paper is that the provision of long-term put options is consistent with declared policy in developing a well-functioning carbon market. The UK Government (or its agent) is heavily involved in many markets including that for its own short and long-term bonds where the risks of these markets have to be assessed in the pursuit of wider policies. Given the declared importance of carbon emission/reductions the proposed Government role is appropriate. It is noteworthy that, as well as having long-standing expertise in financial markets the Government now has substantial and relevant expertise in negotiating carbon saving contracts in the form of the CCAs discussed in Section 1 above. Given a lead from the Government it is quite plausible that other option writers will subsequently emerge.

7. Conclusions

This paper has proposed and discussed a scheme for reducing the market risks involved in long-term investments for reducing carbon emissions (such as carbon capture and EOR/sequestration in North Sea oil fields). The scheme consists of the writing of long-term put options for carbon allowances by the Government or its agent. Such a scheme is consistent with the Government's

[10] Though they can be of 2-3 years duration (such as Long-Term Equity Anticipation Securities (LEAPS). See J.C. Hull (2000).

declared aim of making the value of carbon the centre-piece of its policy framework. While novel, it is argued that, given the objectives and attitudes of the Government and investors, there is scope for the conclusion of such contracts. They offer a low cost route by which the Government can promote its objectives in this area.

Appendix: Calculating the Expected Present Value of the Net Cost of a Put Option

Suppose that the Government sells a European put option for 1 tonne of carbon emissions with an exercise price of K. Let X^* denote the net expenditure or net cost to the Government caused by the optimal exercise of the option at the expiration date. The size of X^* depends on the market price of 1 tonne of carbon emissions at the expiration date, P^*. Equation (1) in the text describes how X^* depends on the market price. Let T denote the number of time periods between the present and the expiration date of the put option.

As noted in the text of the paper, the expected present value of X^* depends on K, T, and the riskless interest rate, r. From the perspective of the present, the market price P^* is uncertain, and the expected present value of X^* also depends on the probability that P^* takes on particular values. Suppose first that $0 \leq \lambda \leq 1$ denotes the probability that there is no market for carbon emissions at the expiration date of the option. Perhaps the market for carbon has collapsed or perhaps it has been superseded by some other form of regulation. In the absence of a market for carbon, the owner of the option who wishes to sell a reduction in carbon emissions must exercise the option and sell at a price K to the Government. The important special case where investors and Government decision makers are confident that a market for carbon will exist at the time the option expires is captured by setting $\lambda = 0$.

We assume that the probability that a well functioning market for carbon exists at the expiration date is given by$1 - \lambda$. Conditional on the existence of a market for carbon, let $f(P^*)$ denote the probability density function which describes the probability that the market price takes on values between 0 and ∞.

A leading candidate for $f(P^*)$ is the lognormal distribution which is often used as a model for the prices of financial assets. If the probability density for P^* is lognormal, then $f(P^*)$ is given by the following equation:

$$f(P^*) = \frac{1}{P^* \sigma \sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2} \left(\ln P^* - \mu \right)^2 \right] \quad \text{(A.1)}$$

where μ is a parameter, σ is a positive parameter, and $\ln x$ denotes the natural logarithm of x. If $f(P^*)$ is given by equation (A.1), then, conditional on the existence of a market for carbon emissions, the mean m and the standard deviation s of P^* are given by the following equations. (See, for example, Mood, Graybill and Boes (1974).)

$$m = \exp\left[\mu + \frac{\sigma^2}{2} \right] \quad \text{and} \quad s^2 = \exp\left[2\mu + 2\sigma^2 \right] - \exp\left[2\mu + \sigma^2 \right] \quad . \quad \text{(A.2)}$$

The above equations can be solved to obtain the parameters μ and σ as a function of m and s.

Figure A.1 shows examples of the lognormal density for the market price P^* for selected values of the parameters m and s. The solid curve in Figure A.1 shows the lognormal density when the standard deviation of the market price, s, equals

€ 20. The dashed curve in Figure A.1 shows the lognormal density when s = €40. For both curves, the mean of the market price is set equal to m = € 30.[11]

Using equation (1) in the text and the notation developed above, the expected present value of the net cost of selling a put option, EPV(X*) , can be written in the following form.

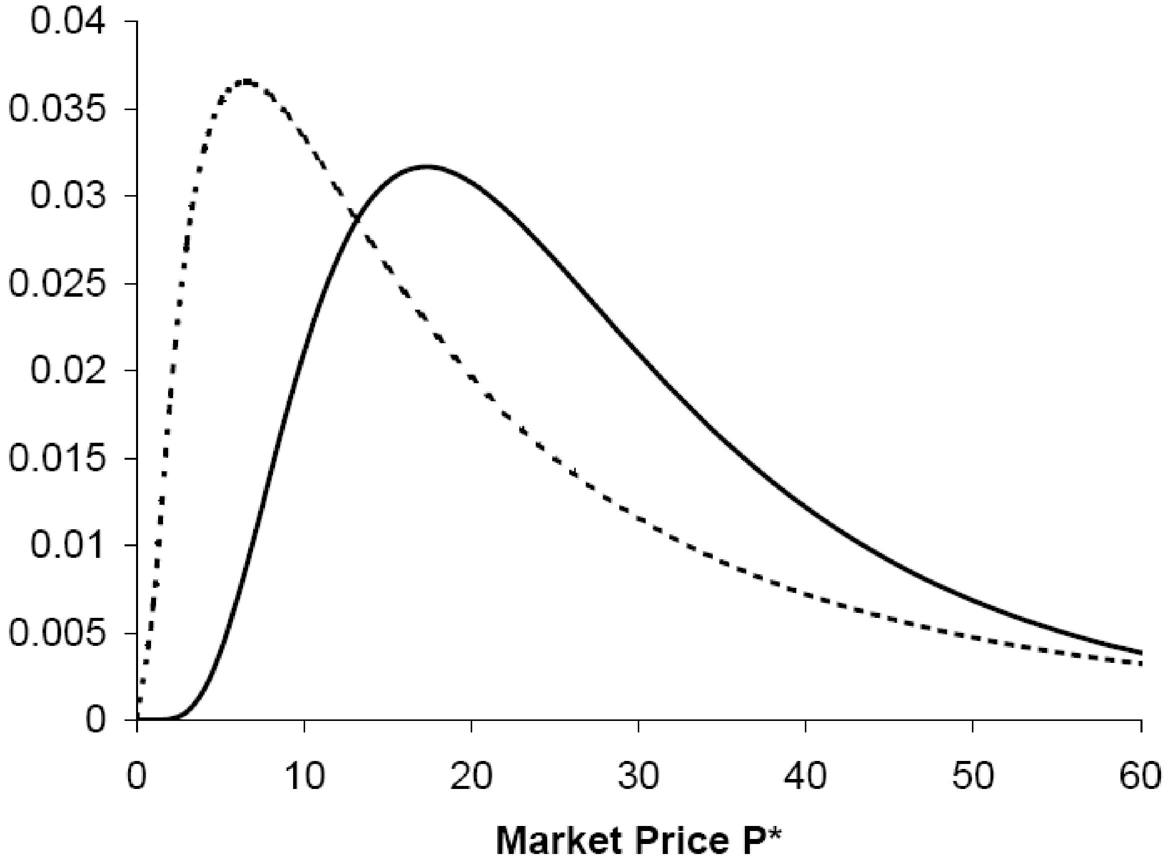
$$\mathrm{EPV}(X^*) = \frac{\lambda K + (1-\lambda)\int_0^K \left(K - P^*\right) f(P^*) dP^*}{(1+r)^T} \tag{A.3}$$

For a specific probability density, $f(P^*)$, the definite integral in equation (A.3) can be evaluated numerically using a mathematical software package. Note that if $\lambda = 1$, then the formula for EPV(X*) in equation (A.3) reduces to the upper bound in equation (3) of the text.

As an example, table A.1 reports the results, for selected parameter values, of using Matlab to evaluate the right-hand side of equation (A.3) assuming that the uncertain price of carbon emissions at the time the option expires is described by a lognormal probability density.

[11] Initially, it may seem surprising that the peak of the density with a standard deviation of € 40 occurs at a smaller value of P^* than the peak of the density with a standard deviation of € 20. An increase in the standard deviation generally increases the size of the upper tail of the density. An increase in the upper tail requires a compensating increase in the probability mass assigned to “lower values” of P^* to keep the mean price equal to € 30.

Figure A.1 Lognormal Densities for Market Price P*
mean = € 30, standard deviation =
€ 20 (solid curve) and € 40 (dashed curve)



In table A.1, it is assumed that the exercise price of the option, K, is equal to the mean, m, of the price for carbon emissions at the expiration date, and that both are equal to € 30 per tonne. That is, $K = m =$ € 30. The annual riskless interest rate is assumed to be $r = .05$. The probability that no market for carbon emissions exists at the expiration date is taken to be $\lambda = 0.2$.

The expected present value of the net cost to the Government of the option, EPV(X^*) , is reported for European put options that expire in 5, 10, and 20 years. That is, $T = 5, 10, 20$. To illustrate the effect of changing the probability density of the market price at the expiration date, the results are reported for two values of the standard deviation of the price of carbon at the expiration date,

s = €20 and s = € 40. The quantity EPV(X*) is reported in euros. For comparison, the upper bound on the cost to the Government in equation (3) in the text is also reported in the last column of table A.1 for selected values of T.

Notice that the expected present value, EPV(X*), and the upper bound, C_0^{MAX}, reported in table A.1 decline as T increases. Because of discounting, the present value of the future cost will typically decline as the number of years until the option expires increases.

Note also that the expected present values of the future cost reported in table A.1 are higher for s = € 40 than for s = € 20. For prices for carbon that are less than the exercise price of the option, K , Figure A.1 suggests that the probability density with s = € 40 puts more probability weight on lower values of P^* than the probability density with s = € 20. Since the future cost incurred by selling the option is higher when the price P^* is lower, it is not surprising that the expected present value of the future cost is higher for a density that puts greater weight on lower values of P^* .

Table A.1 The expected present value, EPV(X*) , of the future cost to the Government from selling a European put option, for selected values of the time to expiration of the option, T , and the standard deviation of the price of carbon emissions at the time the option expires, s. The exercise price of the option, K, and the mean of the distribution of possible prices at the time the option expires, m, are both set equal to € 30. For each value of T, an upper bound on the expected present value, C_0^{MAX} , is also reported.

	s = € 20	s = € 40	C_0^{MAX}
T = 5 years	€ 9.18	€ 11.97	€ 23.51
T = 10 years	€ 7.19	€ 9.38	€ 18.42
T = 20 years	€ 4.42	€ 5.79	€ 11.31

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